Why use Radians?

Below are six reasons why radians should be used to measure angles rather than degrees. There are questions at the end of each section.

1. Pi is great

There is nothing special about 360° (in fact, people at other times in history have used a full circle being 100° or 400°). However, the concept of π is at the centre of maths.

(a) Find the size of a right angle if a full circle is chosen to be 360°, 100°, 400° or 2π radians

2. Rotation speed

Since one radian is the angle that produces an arc length of one radius, the radian measure ties together angle and length, which can make some calculations much easier.

For example, to find the speed of a vehicle in metres per second you can just multiply the radius by the number of radians it turns per second. So if a wheel has radius of two metres and turns at six radians per second the speed is 12 metres per second.

(a) Find the speed in m/s of a wheel with radius 0.5 m turning at 20 *radians* per second

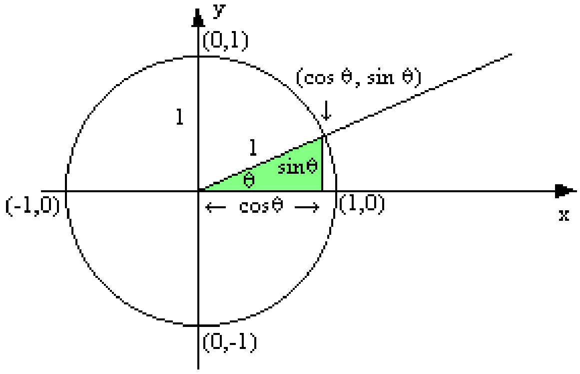
(b) Find the speed in m/s of a wheel with radius 0.5 m turning at 2000 *degrees* per second

(c) Find the speed in m/s of a wheel with radius 0.5 m turning at 10 *rotations* per second

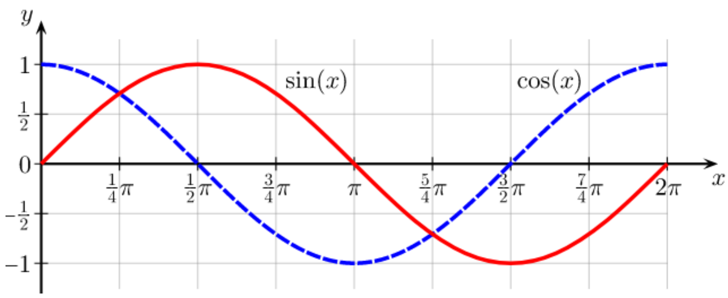
3. Drawing graphs

The unit circle is a circle of radius 1, centred on the origin at (0,0).

Imagine a point starting at (1,0) and orbiting the unit circle anticlockwise. The angle starts at 0 and increases. The vertical height and horizontal width change with the angle. The *trigonometric functions* $sin(\Theta)$ and $cos(\Theta)$ measure this height and width.



The graph below shows the values of $sin(x)$ and $\cos(x)$ as the angle changes:



If drawn in degrees the graphs must be hugely stretched to fit in 360 degrees

(a) Sketch the graphs above, using degrees instead of radians

(b) Cover up the graph above and try and draw it just from considering a point orbiting the unit circle anti-clockwise, and keeping track of its height ($sin(x)$) and width ($co\,s(x)$).

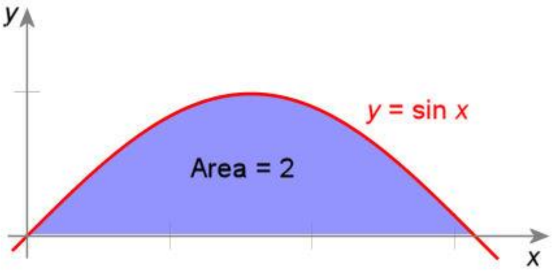
4. Calculus

There is a relationship between the sine and cosine graphs and their gradients

- The gradient of the sine function at any point is equal to the value of the cosine function
- The gradient of the cosine function is equal to the negative of the value of the sine function

There is also a nice formula for the area of the *lobes* of the cosine and sine function

- The area of each lobe is exactly 2.



These formulas only work when the angles are counted in radians, and not degrees

Both the gradient and area calculations rely on the following fact, only true in radians

$$\lim_{\Theta \to 0} \frac{\sin(\Theta)}{\Theta} = 1$$

(a) Find the gradient of the sine graph when $x = \frac{\pi}{2}$ from considering the graph

(b) Find the gradient of the sine graph when $x = \frac{\pi}{2}$ from considering the unit circle

(c) Calculate $\frac{\sin(x)}{x}$ for $x = 0.1$, $x = 0.01$ and $x = 0.001$
Make sure your calculator is in radians mode

5. Sine expansion formula

The sine function (and the cosine function) can both be represented as infinite polynomials:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

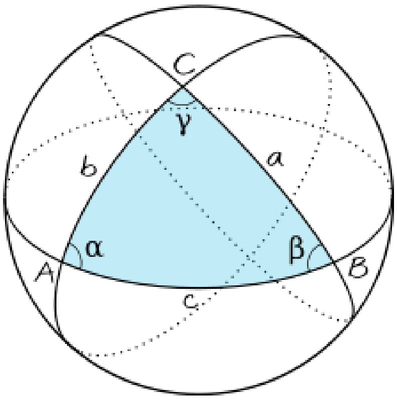
These are only true when x is measured in radians

(a) Evaluate $\sin(0.1)$ using your calculator, and the formula above up to the term $\frac{x^7}{7!}$.
Make sure your calculator is in radians mode

(b) Evaluate $\cos(0.1)$ using your calculator, and the formula above up to the term $\frac{x^6}{6!}$.
Make sure your calculator is in radians mode

6. Spherical geometry

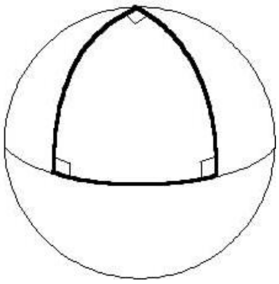
If a triangle is drawn on a unit sphere (a sphere of radius 1) the angles will add up to at least 180 degrees



If angles are measured in radians there is an exact relationship between the area of the triangle (A) and the sum of the angles α, β, γ

$$A = \alpha + \beta + \gamma - \pi$$

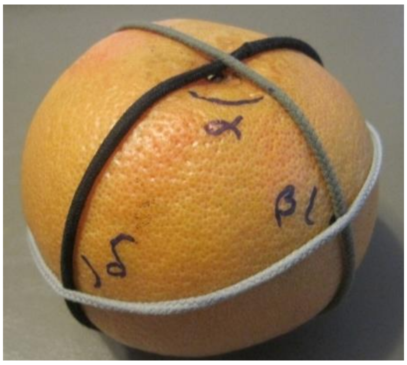
(a) Find the area of the triangle below, made up of three right angles on a unit sphere



(b) Find the area of the triangle above, using the fact the surface area of an entire sphere is

$$S = 4\pi r^2$$

(c) The grapefruit below is approximately a sphere with radius 10 cm.



The angles are $\alpha = 130^{\circ}$, $\beta = 90^{\circ}$, $\delta = 80^{\circ}$
Find the area of the triangle.