Measurements and Uncertainties

Measurement	Instrument	Resolution	Uncertainty or precision
Length	Metre rule	1 mm	±1 mm
	Vernier caliper	0.1 mm	±0.1 mm
	Micrometer	0.01 mm	±0.01 mm
Time	Manual stopwatch	0.01 s	±0.2 s (reaction time)
	Electronic timer	0.01 s	±0.005 s
Mass	Top-pan balance	0.01 g	±0.005 g or ±0.01g
	Spring balance	1.0 g	±0.5 g
Temperature	Mercury thermometer	0.1°C	±0.05°C (or ±0.1°C)

The ***uncertainty*** of a result is the interval within which the true value can be expected to lie.

The ***absolute uncertainty*** of a reading is no smaller than plus or minus half of the smallest division. The absolute uncertainty of a measurement, where two judgements are required (e.g measuring a length using a ruler), is twice this. For multiple readings, the absolute uncertainty is half the range. Absolute uncertainties have the same units as the quantity.

$$\text{absolute uncertainty} = \frac{\text{range (largest value } - \text{ smallest value)}}{2}$$

All measurements should be written as mean value ± measurement error (a ± Δa). E.g. A voltmeter gives a reading of 1.70 ± 0.01 V.

Quoting results along with errors:

- When giving results in terms of scientific notation or in standard form, always quote the value and the error with the same exponent.
- Quote the result to the same number of significant figures as the quoted error implies.
- Always quote the error to 1 or at most 2 significant figures.

1. A stopwatch that is accurate to 100th of a second is used to record timings of an object in motion. What is the resolution of the stopwatch and what would be a typical value for its precision?
2. A metre rule is being used to determine the vertical height of an object. Give two precautions that should be taken to ensure an accurate result.
3. What device can be used to measure widths typically less than a centimetre and what is the precision of such a device?
4. A measured value of 132 is quoted with an uncertainty of 18. Write the value to 2 s.f. along with the uncertainty.

5. A measured value of 11.448 is quoted with an uncertainty of 0.25. Write the value to an appropriate degree of accuracy along with the uncertainty.
6. The potential difference measured on a digital voltmeter is 3.36 V. Give this value together with the instrument uncertainty.
7. A current is measured with an analogue ammeter using a scale from 0 to 5 A. The reading obtained is 4.25 A and the interval size is 0.2 A. Give the value on the ammeter together with the uncertainty.
8. A metre rule is used to measure the width of a bench. The ruler's smallest interval is 1 mm and the length of the bench is measured to be 64.5 cm. Express this length together with the uncertainty in metres.
9. A set of measurements for the diameter of a piece of wire is made and the results are shown in the table.

Diameter (mm)	5.01	4.94	4.98	4.92	4.95

   a) What is the name of the device used to measure such small distances? Give both the value of the resolution and the precision of this device.
   b) What precautions should be taken before using this device?
   c) The true value is 4.81 mm. Explain whether the results are accurate and/or precise.
10. A thermometer is used to record the temperature of water as it is heated from frozen. The results are shown in the table.

Temperature (°C)	1.0	2.4	3.9	5.0	6.3	7.1	7.9	9.6	10.1
Time (min)	0	1	2	3	4	5	6	7	8

a) What is the resolution and uncertainty in the measuring device?
b) Draw a graph and plot the results of temperature (y-axis) against time (x-axis).
c) Draw the best line of fit through the points.
d) Given what you found in part a), determine the nature of the results in terms of random and/or systematic errors and justify your conclusion.

11. A steel rule is being used to measure the length of a metal bar that has a "true" length of 795 mm. The rule can be read to the nearest millimetre. Repeated measurements give the following results.

Reading	1	2	3	4	5
Value (mm)	792	791	791	792	792

a) What is the mean value for the length?
b) Are the readings accurate to 1 mm? Give a reason for your answer.
c) Are the readings precise to 1 mm? Give reasons for your answer.

The ***fractional uncertainty*** is the absolute uncertainty divided by the measured value (if multiple readings, divided by the mean).

$$\text{fractional uncertainty} = \frac{\text{absolute uncertainty}}{\text{mean value}} = \frac{\Delta a}{a}$$

This can be converted into a **percentage uncertainty** by multiplying by 100.

$$\frac{\Delta a}{a} \times 100 = \varepsilon a$$

There are different rules for combining uncertainties. Remember:

Δa = absolute uncertainty

εa = percentage uncertainty

When adding or subtracting data with uncertainties, add the absolute uncertainties.

When multiplying or dividing data with uncertainties, add the percentage uncertainties.

When raising data with an uncertainty to a power, multiply the percentage uncertainty by that power.

Combination	*Uncertainties*
$a = b \pm c$	$\Delta a = \Delta b + \Delta c$
$a = bc$ or $a = b/c$	$\epsilon a = \epsilon b + \epsilon c$
$a = b^c$	$\epsilon a = c \times \epsilon b$

When multiplying data with an uncertainty by a constant, multiply the absolute uncertainty by that constant but not the percentage uncertainty.

1. A thermometer is graduated in intervals of 1°C. What is the measurement uncertainty associated with this thermometer?
2. What is meant by the resolution of an instrument?
3. If the resolution of a set of weighing scales is said to be 0.1 g, what is the uncertainty in the values obtained?
4. An analogue ammeter is graduated in intervals of 0.2 A. What is the uncertainty of the device for recording current?
5. If the value of a measurement is a, what does Δa mean?
6. How is the percentage uncertainty determined from a single measurement whose value is a?
7. How is the absolute uncertainty determined from a range of measurements?
8. A particular resistor was measured on five occasions to give the following results: 1.20 kΩ, 1.16 kΩ, 1.24 kΩ, 1.22 kΩ and 1.28 kΩ. What is the mean value of the resistor?
9. In the above set of results, what is the uncertainty associated with the measuring device used?
10. In question 8, what is the absolute uncertainty in the measurement?
11. In question 10, what is the percentage uncertainty in the measurement?
12. The resistance of a component is being measured. The potential difference across it is 8.2 ± 0.2 V and the current through it is 0.8 ± 0.1 A. The resistance, R, of any component is given by the equation $V = IR$, where V is the potential difference and I is the current.
    a) What is the value of the resistance of the component?
    b) Determine the percentage uncertainties in both the potential difference and current readings?
    c) From part b) calculate i) the total percentage uncertainty in R and ii) the absolute uncertainty in R.
    d) Give the final value of the resistance together with its uncertainty.

13. The density of a piece of metal in the shape of a cube is being determined. The mass of the cube is measured to give the following results: 34.5 g, 34.2 g, 34.7 g, 34.9 g and 34.1 g.

a) Calculate the mean mass of the metal cube. Give your answer to an appropriate number of significant figures.
b) What is the uncertainty in the weighing scales used to determine the mass?
c) Determine the absolute and percentage uncertainty in the above set of measurements and give your answer in the form: mass ± uncertainty in the mass.

The dimension of the cube is 2.3 ± 0.01 cm for each side.

d) Determine the volume of the cube and calculate the percentage and absolute uncertainty in the volume of the cube.
e) Density is given by $\rho = m/V$. Calculate the absolute uncertainty in the density of the metal and give your final answer in units of kg/m^3.

14. Hooke's law states that the extension of a spring is directly proportional to the load, i.e. $F = kx$ where F is the load in N, x is the extension in m and k is a constant, known as the spring constant.

a) If the spring extends by 4.6 mm when a load of 15 N is applied, determine the value of the spring constant in N/m.

The uncertainty in the extension is ± 0.5 mm and the uncertainty in the force is ± 0.5 N.

b) Calculate the percentage uncertainties in i) the extension and ii) the load.
c) Determine the absolute uncertainty in the spring constant and write your answer as spring constant ± uncertainty.

Other measurements taken using the same spring give a set of spring constants of values 3300, 3240, 3190 and 3140 N/m.

d) Using the result in part c) together with the four other results above, determine the mean spring constant and the measured uncertainty in this set of results.